世新大學九十七學年度碩士班招生考試試題卷

系所組別	考 試 科 目
資訊管理學系資訊科技組	離散數學

※本考題 □可使用 ☑禁止使用 簡易型電子計算機

※考生請於答案卷內作答

共十題，每一題 10 分

1. (5 points each)
   (a) How many ways can we arrange the letters in the "MATHEMATICS"?
   (b) How many 16-bit words contain exactly six 1's?

2. Prove that there are infinitely many primes.

3. Let $G(V, E)$ be a loop-free connected planar graph with n vertices, $m > 2$ edges, and r regions. Based upon the result of Euler's Theorem: $n - m + r = 2$, please show that $3r \le 2m$ and $m \le 3n - 6$.

4. Show a combinatorial proof for $\binom{n+2}{r} = \binom{n}{r} + 2\binom{n+1}{r-1} + \binom{n}{r-2}$, where $n \ge r \ge 2$.

5. Prove that $\sum_{j=1}^{m} j2^{j-1} = 1\cdot 2^0 + 2\cdot 2^1 + 3\cdot 2^2 + \ldots + m\cdot 2^{m-1} = (m-1)\cdot 2^m + 1$ by Mathematical Induction.

6. Let $A = \{1, 2, 3, \ldots, 32, 33, 34\}$, i.e., A contains the integers ranging from 1 to 34, and H be any subset of A with $|H| = 18$. Prove that there must exist $x, y \in H$ such that the sum of x and y is 35. (Hint: You may use the Pigeonhole principle.)

7. Suppose that $B = \{x, a, b, c, d\}$. Please answer the following questions.
   (a) How many closed binary operations on B have d as the identity?
   (b) Among the closed binary operations of part (a), how many are commutative?
   (c) How many closed binary operations on B have an identity?

請接背頁

世新大學九十七學年度碩士班招生考試試題卷

系所組別	考 試 科 目
資訊管理學系資訊科技組	離散數學

※本考題 □可使用 ☑禁止使用 簡易型電子計算機

※考生請於答案卷內作答

共十題，每一題 10 分

8. (5 points each)

(a) Derive a recursive definition for $S_n = 1 + 2 + \ldots + n$，$n \geq 1$.

(b) Assume that $n = 2^k$, for some positive integer k. Solve the recurrence relation $T(n) = 2 \cdot T(\frac{n}{2}) + n$, where $T(1) = 1$.

9. Suppose Z denotes the set of all integers. Let the relation Π on $Z \times Z$ be defined by the rule $(x, y)\ \Pi\ (z, w)$ if $x + w = y + z$.

(a) Prove that Π is an equivalence relation.

(b) Show the equivalence classes of Π in the plane.

10. Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Given $A = \{5, 10\}$, $B = \{2, 3, 4, 6, 7\}$, $C = \{6, 7, 8, 9, 10\}$, and $D = \{3, 6, 9\}$. Please compute

(a) $\overline{A \cap B}$

(b) $(D - C) \cup \overline{B}$

(c) $\overline{C} - A$

(d) $A \times (B \cup D)$

(e) $D \times A$