I. (12%) Find the domain of the function

\[ f(x) = \frac{x + 2}{x^2 - 1} \]
\[ f(x) = \frac{x^4}{x^2 - 6} \]
\[ g(x) = \sqrt{x^2 - 6x} \]
\[ g(x) = 3x^2 + 2x + 1 \]

II. (9%) Find the value of the one-sided limit

\[ \lim_{x \to 1^+} \frac{x + 2}{x - 1} \]
\[ \lim_{x \to 1^-} f(x) \text{ where } f(x) = \begin{cases} 3x & \text{if } x > 1 \\ 4x + 2 & \text{if } x \leq 1 \end{cases} \]
\[ \lim_{x \to 1^-} \frac{x - 1}{x - 4} \]

III. (21%)
3.1 (9%) Find the derivative of the following functions

a. \[ f(x) = (4x^2 + 3x - 1)^{\frac{3}{2}} \]

b. \[ f(x) = \frac{2x}{\sqrt{3x^2 + 1}} \]

c. \[ y = f(x) = \sqrt{3x + 2} \times (9x - 1)^5 \]

3.2 (4%) Let \( H(x) = (f(x))^2 \), with \( f \) differentiable at \( x = 2 \). Find \( H'(2) \) if \( f'(2) = 4 \) and \( f''(2) = 3 \).

3.3 (8%) Find ‘all’ of the second-order partial derivatives of the following functions

a. \[ f(x, y) = x^2 + xy^3 \]

b. \[ f(x, y) = e^{-xy} \]

IV. (16%) Evaluate the definite or indefinite integrals of the following functions.

a. \[ \int \sqrt{2} \, dx \]

b. \[ \int \ln x \, dx \]

c. \[ \int x^2 e^x \, dx \]

d. \[ \int x(x^2 - 1)^3 \, dx \]
V. (12%) Use the trapezoidal rule and Simpson’s rule to approximate the following value of integral. Compare the result with the exact value of the integral.

\[
\int_{0}^{2} x^2 dx \ ; \ n=6
\]

VI. (30%) The total weekly revenue (in dollars) for a firm in producing and selling its products of A and B is given by

\[
R(x, y) = -\frac{1}{4} x^2 - \frac{3}{8} y^2 - \frac{1}{4} xy + 300x + 240y
\]

x is the number of product A and y is the number of product B. The total weekly cost attributable to the production is

\[
C(x, y) = 180x + 140y + 5000
\]
dollars.

(1) Determine how many products of A and B should produce per week to maximize its profit(s), and how much is the profit. Furthermore, please make sure the critical point(s) is a relative maximum, not a relative minimum.

(2) There is a constraint of production and it should be restricted to a total of exactly 230 units \((x + y = 230)\) each week. Under this condition, determine how many products of A and B should produce per week to maximize its profit(s), and how much is the profit.